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Spatial and temporal gradients in the cosmological constant

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Abstract: It is possible that there may be differences in the fundamental physical parameters from one side of the observed universe to the other. I show that the cosmological constant is likely to be the most sensitive of the physical parameters to possible spatial (or temporal) variation, because a small variation in any of the other parameters produces a huge variation of the cosmological constant. It therefore provides a very powerful *indirect* evidence against spatial gradients or temporal variation in the other fundamental physical parameters, at least 40 orders of magnitude more powerful than direct experimental constraints. Moreover, a gradient may potentially appear in theories where the variability of the cosmological constant is connected to an anthropic selection mechanism, invoked to explain the smallness of this parameter. Some suggestions are made about how one may observe such a spatial gradient. In the Hubble damping mechanism, I calculate the possible gradient. While this mechanism demonstrates the existence of this effect, it is too small to be seen experimentally, except possibly if inflation happens around the Planck scale.

Keywords: Cosmology of Theories beyond the SM, Physics of the Early Universe.

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Contents

1. Introduction

Within the Standard Model of particle physics, describing our present best understanding of the fundamental forces and particles underlying all phenomena, there are 19 physical parameters or "coupling constants" [1]. These include the fine structure constant $\alpha = 1/137$ as well as the masses of the quarks and leptons. These are not predicted within the theory, but are free parameters whose value must be extracted from experiment. However, within the theory one feature is clear — these parameters are constants. They do not depend on the time of the year nor our location in space.

Despite this expectation, there exists a continual effort to look for spatial or temporal variation in these parameters [2]. It is certainly worthwhile to check this most basic property of our physical theories. Moreover, as will be discussed below, there exist physical mechanisms such that different parts of the universe may have different values of the physical parameters. These mechanisms have been invoked in cosmology and in attempts to implement anthropic selection mechanisms [3]–[10].

In these applications, the full Universe is much greater than the portion of it that we can observe. In addition there is the unusual feature is that the parameters can vary, perhaps in a continuous fashion, in different regions. More work is needed to construct a fully realized cosmology incorporating these mechanisms, but it does appear physically possible to construct theories in which the parameters could vary in different parts of the universe. At this stage, it is harder to know how we will test such theories. However, a possible test emerges if the variation of parameters is continuous. In this case, we may see a small variation when we look across our observable portion of the universe. Thus we can look for a small continuous gradient in the parameters.

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I will argue that the cosmological constant is likely to be the parameter which is most sensitive to spatial or temporal variations. As a consequence, the cosmological constant provides extremely powerful indirect evidence against significant variation of the other physical parameters, unless that variation occurs in several parameters in a highly fine-tuned fashion.

I will also argue that a physical realization of the anthropic principle [3] leads us to expect that there could be spatial variations in the physical parameters and that such variation would be most sensitively be manifest in the cosmological constant. While it is possible that the effect may be too small to be observed, the opportunity to provide any form of experimental evidence on the anthropic principle makes it worthwhile that this variation be tested.

2. The cosmological constant and its potential variation

The cosmological constant Λ is defined by the vacuum value of the energy-momentum tensor $T_{\mu\nu}$, $\mu, \nu = 0, 1, 2, 3$ via

$$\langle 0|T_{\mu\nu}|0\rangle = \Lambda g_{\mu\nu}\,, \tag{2.1}$$

where $g_{\mu\nu}$ is the metric tensor. Since T_{00} is the hamiltonian, Λ is equivalent to the energy density of the vacuum. It has dimensions of $(\text{mass})^4$. The cosmological constant appears to be visible as an acceleration in the expansion of the universe [12].

There are many different contributions to the cosmological constant. Each particle and each interaction contributes to the vacuum energy. In fact, each particle's contribution is infinite when calculated separately within the Standard Model because it diverges at high energy.

However, the calculation of the high energy portion is unreliable since we do not yet know the correct theory that holds at high energy. Even if the high energy contributions are cut off by new particles or interactions (as happens for example in supersymmetry), contributions remain from those energies where we already understand the physics. Quantum Chromodynamics contributes effects of order $1\,\text{GeV}^4$. The electroweak theory contributes effects of order $(100\,\text{GeV})^4$. If supersymmetry is present in the theory but is broken at a low an energy as is phenomenologically possible, i.e. $1\,\text{TeV}$, then it yields contributions of order $(1\,\text{TeV})^4$. If instead, the divergences are cut off at order the Planck mass, $M_P = 1.2 \times 10^{19}\,\text{GeV}$, then the contributions are of order M_P^4.

The most important feature to be accommodated within any theory of the cosmological constant is the extremely tiny value of the final answer compared to the size of any of the contributions. The presently favored experimental value ($\Omega_\Lambda \sim 0.7$) corresponds to

$$\begin{aligned}\Lambda_0 &= 2.7 \times 10^{-47}\,\text{GeV}^4 \\ &= 2.7 \times 10^{-59}\,\text{TeV}^4 \\ &= 1.2 \times 10^{-123} M_P^4\,. \end{aligned} \tag{2.2}$$

The different contributions must nearly cancel to very many orders of magnitude. The situation is even more difficult than if the experimental value were zero. In that case one might imagine that there exists a mechanism that forces the value to be exactly zero. However, with a non-zero experimental value, this hope must be abandoned and we appear to be faced with *defacto* fine tuning of the different contributions with a tiny but non-zero residual.

If *any* of the contributions may have a gradient, then the overall cosmological constant will also vary. Because of the large cancellation, a small percentage variation in a given contribution leads to a large percentage variation in the total. A function that varies continuously in spacetime is a field, and so in this case at least some parameters are described by the values of one or more fields. There would continue to be contributions to Λ that are truly constant. Let us call these effects Λ_{other}. If the mass scale for relevant underlying physics is M_*, these contributions to the cosmological constant will be of order $\Lambda_{\text{other}} \sim M_*^4$. Added to this will be the contributions due to the variable fields (Λ_{fields}) which would also be of this order and which could occasionally (i.e. in some locations) cancel the other constant value,

$$\Lambda(x) = \Lambda_{\text{other}} + \Lambda_{\text{fields}}(x)\,. \tag{2.3}$$

The picture that we are then led to involves these fields having a range of values in the early universe such that in some region the contributions largely cancel. The cosmological constant would not be strictly a constant but would allow spatial gradients. Temporal variation is already recognized as possible, and is manifest in theories of quintessence [13] However, spatial variation of the cosmological constant is also possible. While locally the value could have little variation, a variation could be observed when comparing the value at different sides of the observed universe.

Because of the enormous cancellation implied in eq. (2.3), a variation of Λ_{fields} of one part in 10^{59} (assuming $M_* = 1\,\text{TeV}$) would lead to a variation in the net cosmological constant of order unity. Thus the search for variations in Λ provides an extremely sensitive probe for variation in the underlying physics influencing Λ_{fields}.

3. Empirical constraints

A gradient in the cosmological constant will provide a"tilt" to the universe. As such, it is one of a class of physical effects that can provide a dipole anisotropy to the universe [14]–[18]. Observations of the cosmological constant favor the spatial regions around $z \approx 1$. It is on this scale that the cosmological constant first has observable consequences. The dynamics of galaxies on much shorter scales is not influenced greatly by the presence of a cosmological constant of the observed magnitude. Anisotropy of signals sensitive to $z \geq 1$ can then potentially probe a gradient in the cosmological constant.

A study of a possible tilt in the distribution of the matter density has been performed by Turner [14]. In this case, the gradient was assumed to be generated by a scalar field. In a matter dominated universe, isocurvature fluctuations of the scalar field led to a difference between the rest frame described by the local matter distribution and that described by the CMB radiation, For curvature mode fluctuations in a matter dominated universe, there was a compensation (from the Sachs-Wolfe effect) such that both matter and the background radiation will flow in the direction of the tilt such that the anisotropy of the cosmic microwave background generated by the tilt would be indistinguishable from the effect of a local peculiar velocity. However, in a universe dominated by dark energy both types of fluctuations lead to an observable tilt. Moreover, the existence of a tilt could be inferred from the comparison of the CMB dipole with that of the anisotropy of other radiation such as the X-ray radiation from objects at high redshifts. Stebbins and Veeraraghavan [18] have analyzed the particular case of a minimally coupled scalar field with vanishing potential in a matter dominated cosmology, and have shown that the CMB quadrupole anisotropy measurements strongly constrain the tilt in this scalar field.

There is presently no convincing evidence of a tilt in the observed universe. It would take an detailed calculation to turn this lack of evidence into a numerical bound on the magnitude of a possible gradient of a cosmological constant. That calculation is beyond the scope of the present paper. In the rest of this section, I will survey some of the relevant recent experimental constraints and also make some suggestions for more direct tests.

The X-ray background is believed to be capable of probing sources located out to $z \leq 5$ [19]. The dipole pattern in the X-ray background is due to two effects: 1) the X-ray sources themselves responding to the large scale structure and 2) our own motion relative to the X-ray background [19]. At present there is no convincing demonstration of a difference between the X-ray dipole and the CMB dipole. The work of Lauer and Postman [20], comparing the motion of the Local Group relative to $z < 0.05$ Abell clusters suggested evidence for bulk motion. This has been disputed by others, while other more recent evidence has emerged [21]–[25]. However there is not an agreement in the direction of the proposed bulk motion. The present status and possible systematic effects are summarized in Scharf et. al. [25]. Note that all of these measurements are presently constrained to values of $z < 0.1$ [25].

Direct studies of the velocity field have been extended out to $z \sim 0.3$ using other techniques [26]–[28]. The use of Type 1a supernova in [26] is particularly interesting in that these measurements are of the same type as were used to identify the presence of the cosmological constant [12]. As we argue below, it would be especially interesting if a similar study of the velocity dipole could be performed at higher redshifts. Overall the conclusions of these studies can be summarized as giving no clear indication of a change in direction of the velocity dipole near $z \sim 0.3$

The direct supernova measurements of the cosmological constant from $z \sim 1$ supernova measurements [12] can provide another way to identify a gradient in that quantity. These measurements have direct access to the value of the cosmological constant as they roughly constrain the combination $0.8\Omega_m - 0.6\Omega_\Lambda$. With enough statistics, the analysis can be performed by separating the data set into different hemispheres (or finer bins) and providing a measurement of the cosmological constant in each hemisphere. The average cosmological constant will be different have different values in hemispheres chosen in line with the tilt in the cosmological constant. If the measurement of the whole-sky sample has an uncertainty σ_w, the standard deviation on the difference of the two hemispheres will be approximately $2\sigma_w$. Present data can therefor provide only a weak constraint on such a gradient. However, the SuperNova/Acceleration Probe (SNAP) [29] aims at a 1% measurement of Ω_Λ, and this can provide a significant constraint on any gradient of the cosmological constant.

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Another possibility for a measurement of a large scale dipole has recently become available through the all-sky mapping of fluctuations in the Cosmic Microwave Background with WMAP [31]. In this data analysis, an overall velocity dipole has been accounted for. However, it is still possible that there could be a dipole in the magnitude of the CMB fluctuations at different l values in the power spectrum. In other words, does the power spectrum look the same in all directions, or might there be a spatial gradient in the magnitude of the CMB fluctuations? The position of the first Doppler peak is determined largely by the geometry of the Universe, but the height is sensitive to the cosmological constant [32]. The CMB fluctuations have been amplified by propagation through the Universe from the time of last scattering. A directional survey of the height of the the first Doppler peak (or of other properties of the fluctuation spectrum) could be performed by computing the power spectrum around the Doppler peak utilizing data from only a portion of the sky centered on some direction, and then seeing if there is any variation in the results as one studies the different directions in the sky. A formalism for the analysis of such variations exists [30]. This study would provide a unique form of evidence on the isotropy of the universe over distance scales far beyond what other direct measurements have probed.

4. Constraints on other parameters

The search for a spatial or temporal variation in the cosmological constant is also an indirect search for the variation of other physical parameters. This is because all the physical parameters influence the cosmological constant in one way or another. Moreover, because the cosmological constant is so exquisitely sensitive, this indirect measurement is far more powerful than direct searches. In this section we explore the sensitivity of the cosmological constant in searches for variations of the other parameters.

The possibility of spatial variations of the physical parameters has been discussed by Barrow and O'Toole [2]. At present there are suggestive but tentative indications for a temporal variation in the fine structure constant of order

$$\frac{\Delta\alpha}{\alpha} \sim 10^{-5} \tag{4.1}$$

from quasar spectral lines [33, 34]. However, such a variation of α by itself would have an enormous effect on Λ. The fine structure constant influences the cosmological constant indirectly through its coupling to charged particles. Each charged particle receives a shift in its mass due to electromagnetism, and the masses subsequently influence the zero-point energy. Therefore, changing α changes the energy of the vacuum state.

When calculated strictly within QED, this change is infinite, since the electromagnetic self energy diverges. However, this divergence is only logarithmic, and hence is not deeply sensitive to the high energy scale of the complete final theory. We will proceed in a fashion motivated by effective field theory. The divergence comes from high energy where we do not know the correct theory. A somewhat conservative approach is to calculate only that portion of the effect that comes from the energy scales that we have already probed experimentally. This portion of the answer will be unchanged by the introduction of new particles or interactions at high energy. New physics will introduce new and different effects. While there appears to have been a cancellation in the total magnitude of the cosmological constant, this would not imply that there would be independent cancellations in the variation of Λ with respect to each of the basic parameters. We have no reason to expect that the net magnitude of the α dependence will be smaller than the effects that we calculate here. Therefore the estimate below can be treated as rough lower bounds on the sensitivity of the cosmological constant to the variation in α.

At present, we have the correct theory at least up to the scale of the weak interactions, which can be taken to be approximately the Higgs vacuum expectation value $v = 246\,\text{GeV}$. Let us therefore estimate the effect of changing α due to radiative corrections below this scale.

The influence of the fine structure constant on Λ can be estimated via

$$\frac{\delta\Lambda}{\delta\alpha} = \sum_i \frac{\delta\Lambda}{\delta m_i}\frac{\delta m_i}{\delta\alpha}\,, \tag{4.2}$$

where the sum is over all particles. Both variations are greatest for the most massive charged particle, which by far is the top quark ($m_t \sim 175\,\text{GeV}$), so we can confine our investigation to to that particle uniquely. In perturbation theory the variation of the top quark mass due to electromagnetic effect below the scale v is given by the self energy

$$\left.\frac{\delta m_t}{\delta\alpha}\right|_{E<v} = \frac{m_t}{2\pi}\ln\frac{v^2+m_t^2}{m_t^2}\,. \tag{4.3}$$

Similarly the leading low energy contribution of the top quark to the vacuum energy is

$$\Lambda|_{E<v} = 2\int^v \frac{d^3k}{(2\pi)^3}\frac{1}{2}\omega_k\,, \tag{4.4}$$

where the extra factor of two is for the two different spin states. This leads to an estimate

$$\begin{aligned}\left.\frac{\delta\Lambda}{\delta m_t}\right|_{E<v} &= 2\int^v \frac{d^3k}{(2\pi)^3}\frac{1}{2}\frac{m_t}{\omega_k} \\ &= \frac{m_t}{4\pi^2}\left(v\sqrt{v^2+m_t^2} - m_t^2\ln\frac{v+\sqrt{v^2+m_t^2}}{m_t}\right).\end{aligned} \tag{4.5}$$

Combining these we get

$$\begin{aligned}\left.\frac{\delta\Lambda}{\delta\alpha}\right|_{E<v} &= \frac{m_t^2}{8\pi^3}\left(v\sqrt{v^2+m_t^2} - m_t^2\ln\frac{v+\sqrt{v^2+m_t^2}}{m_t}\right)\ln\frac{v^2+m_t^2}{m_t^2} \\ &\sim 10^{52}\Lambda_0\,,\end{aligned} \tag{4.6}$$

where Λ_0 is the present value of the cosmological constant quoted in eq. (2.2). Note the extreme sensitivity — a fractional change in α of order 10^{-5}, in isolation, would yield a

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cosmological constant 47 orders of magnitude larger than allowed! Conversely, the lack of large variation in the cosmological constant would constrain the variation in α in isolation to be less than less than one part in 10^{47}.

Similar estimates can be given for the effects of variation of other physical parameters. For example the same methods lead to the effect of varying the electron mass

$$\begin{aligned} m_e \frac{\delta\Lambda}{\delta m_e}\bigg|_{E<v} &= \frac{m_e^2 v^2}{8\pi^2} \\ &= 10^{42}\Lambda_0 \end{aligned} \tag{4.7}$$

and the Higgs vacuum expectation value itself,

$$\begin{aligned} v\frac{\delta\Lambda}{\delta v}\bigg|_{E<v} &\approx m_t \frac{\delta\Lambda}{\delta m_t}\bigg|_{E<v} \\ &= 10^{52}\Lambda_0\,. \end{aligned} \tag{4.8}$$

We see that all of these variations are highly constrained by even the roughest lack of variation of the cosmological constant.

These estimates are in reality very crude since we are neglecting the physics beyond the electroweak scale. This can easily be more important than the physics which we do consider and could overwhelm the result presented above. However, there is no reason to expect that physics from higher energy would lead to a smaller residual than that which we estimate.[1] Given these caveats, it is important to treat the result not as a specific bound on the variation of α, but as a qualitative lesson. We have learned that the variation of the fundamental parameters can have a tremendous influence on the cosmological constant and that, unless some special situation occurs, the information on the cosmological constant already rules out the magnitudes of variation that can be tested in direct searches.

The caveat within the previous sentence, about special situations, is needed because there is a logical possibility that the variation of the parameters occurs in such a way that conspires to leave the cosmological constant unchanged. If one considers the space of all possible values of the physical parameters (which in the Standard Model is a 19 dimensional space), then there is a surface in that space corresponding to a fixed cosmological constant. Along that surface the variation due to one of the parameters is compensated by the effects of changes in the other parameters. It is logically possible that the only variations of the parameters allowed are such correlated variations. For this reason, direct searches do give independent information compared to the indirect constraints described in this section. However, there is no known or suspected reason for allowing only such correlated and fine-tuned variations.

The previous considerations are also relevant for possible temporal variation of the fundamental parameters. A time variation of the fine structure constant of the order being

[1]Recall that what is being studied here is not the overall cosmological constant but the the variation of Λ with respect to the other parameters. Even if we are faced with an apparent cancellation in the overall magnitude, it would take separate "miracles" if there were large cancellations in the variation of Λ with respect to each of the other parameters.

discussed in the literature [33, 34] would lead to an enormous variation of the cosmological constant. While it is possible that the cosmological constant was quite different in the extremely early universe, it could not have been very different in the times that can be explored by direct observation through looking at distant (older) objects. Significant temporal variation can only occur if other parameters are also variable in a highly fine-tuned fashion. Note that although one accesses temporal variation by looking at distant objects, it can be differentiated from spatial variation. A time variability appears the same in all directions while a spatial variation would be different in different directions.

5. Anthropic gradient in the cosmological constant

In this section, I argue that the mechanism which allow the anthropic selection of a small cosmological constant may potentially leave a residual effect of a gradient in the cosmological constant.

The anthropic principle [3] can be motivated by the observation that the physical parameters are, fortunately, almost uniquely favorable for the existence of life. Very small changes in the parameters would lead to a situation incapable of life. This can be true whether one uses a narrow definition of life ("life as we know it"), or much broader definitions. As an example of the first type, slight changes in the masses of the up and down quarks and/or electron would lead to a world with no stable protons nor hydrogen atoms, which precludes all organic chemistry as we know it. However, heavier atoms could still exist, so perhaps a different form of life would be possible. In contrast, a variation of a different sort, a slight increase of the Higgs vacuum expectation value (with the other parameters held fixed), would lead to the instability of all elements except hydrogen [4] and would lack the complex chemistry which is likely needed for *any* form of life. Similarly, a variation in the cosmological constant, small on its natural scale, would lead to a Universe where no matter clumps gravitationally, which would again be sterile [5]. Given the potentially allowed range of the parameters in the Standard Model, the anthropically allowed region is a very tiny part of the parameter space. This leads to the hypothesis that the observed values of the parameters are connected to the existence of life, which is a general statement of the anthropic principle. There are physically reasonable situations where the anthropic condition is a natural constraint. This occurs, for example, if there are multiple regions in the universe that can have different values of the parameters. In this case, it is a natural consequence that we would only find ourselves in one of those regions where the parameters were amenable for life. The problem of the cosmological constant is so severe that it motivates us to take seriously a multiple domain anthropic "solution" [5] and [7]–[9].

If a theory can allow multiple values of the physical parameters, the variation of those parameters could be continuous or discrete. If they are discrete, then the residual gradient that is discussed in this paper would not occur. However, it is equally plausible that the variation could be continuous. A theory with a continuously variable parameters could naturally satisfy the anthropic constraint. The only requirement is that both signs of the cosmological constant be possible. In this case, there will be a region where the value goes through zero and in this neighborhood arbitrarily small values are possible. For this region to become our observed universe, we would need to invoke an inflationary epoch in which the region greatly expands. However, inflation [35] is likely needed in any case, so that this is not a difficult additional requirement. In the early universe, gradients in the parameters would naturally exist. Inflation would smooth these out, but some residual variation might exist. Thus we see that anthropic arguments concerning the cosmological constant also provide a motivation for a spatially varying Λ.

There has been little work on mechanisms for variable parameters. Two mechanisms are known, one involving scalar fields with very flat potentials [7]–[9] and another involving four-form fields [8]–[10] such as appear in string theory. I will provide an estimate of the spatial variation in Λ within the former mechanism.

6. Example: Hubble damping

Let us now study the particular mechanism corresponding to a light scalar field with a very flat potential. This was explored in more detail in ref [7, 9]. Note that this scalar field is not the inflaton and is not coupled to other forms of matter. We will see that this mechanism necessarily does lead to fluctuations in the cosmological constant across our visible universe, although the magnitude of the effect will be small unless inflation takes place close to the Planck scale.

First we recall some results from previous work. The most important is that, in order to not be evolving significantly today, the potential must be very flat, satisfying

$$V'(\phi) \leq 10^{-122} M_P^3 \,, \tag{6.1}$$

where the prime denotes the derivative with respect to ϕ. Because this is so small, the field ϕ must itself be very large $\phi \geq 10^{58} M_P$ in order to influence the cosmological constant. In turn, the large value of the field can only be accomplished, while naturally keeping the gradient energy small, if there are extremely many, $N \geq 10^{148}$, e-foldings of inflation, such as occurs in eternal inflation. This latter requirement comes from the fact that the field ϕ exhibits a random walk in its magnitude during inflation, with a quantum fluctuation of magnitude $\Delta\phi \sim H/2\pi$ in a region of size H^{-1} during each e-folding time of inflation [11]. These fluctuations allow ϕ to grow in magnitude in order to satisfy the constraint $\phi \geq 10^{58} M_P$.

It is the quantum fluctuations of ϕ that will also generate fluctuations in the cosmological constant. The logic is as follows. The key is that the field ϕ has random fluctuations of size H per e-folding time, H^{-1}, during the de Sitter phase of inflation [11], but these are then effectively frozen in the subsequent radiation dominated and matter dominated phases after inflation. The regions that we see at opposite sides of the observable universe can have somewhat different values of the field ϕ because the field fluctuations have been randomly different during the inflationary period. The field fluctuations in turn will lead to fluctuations in the cosmological constant.[2] To estimate these fluctuations we must go through three steps. First we must trace back our present universe to the time at the end of inflation in order to find out how many different regions of size H^{-1} there were in the volume that led to our visible universe. Secondly, we need to see how different the values of ϕ could have have been in those regions. We do that by tracing the regions back through the deSitter phase until regions from opposite sides of the universe were in causal contact in a volume of size H^{-3}. This will allow us to estimate the number of e-foldings that occurred from the time of causal contact until the end of inflation, and hence will provide an estimate of how different the field values can be in the different regions. The last and most uncertain step is to transform the fluctuations in ϕ into fluctuations in Λ.

The observable universe today consists of very many regions that were spatially disconnected at the end of the inflationary period. Consider an initial region of size H^{-1} at the end of inflation. This will have expanded by a factor

$$\frac{a_{\text{now}}}{a_{\text{initial}}} = 10^{32}\frac{T_r}{M_P} \tag{6.2}$$

in the subsequent time. This formula emerges from tracing backwards the evolution of the scale factor from the present time into the epoch when it was radiation dominated [37]. Here T_r is the initial reheating temperature after inflation. With efficient conversion of the energy density driving inflation into reheated matter we have

$$H^2 \sim \frac{T_r^4}{M_P^2}\,. \tag{6.3}$$

Combining these factors, the initial patch has grown to a size

$$R_{\text{now}} \sim \frac{a_{\text{now}}}{a_{\text{initial}}}H^{-1} \sim \frac{10^{32}}{T_r} \tag{6.4}$$

now. Despite the large growth this is still very small compared to the size of the universe

$$R_{\text{universe}} \sim 10^{26}\text{meters} \sim \frac{10^{61}}{M_P}\,. \tag{6.5}$$

Thus the present universe consists of

$$N_{\text{regions}} \sim \left(\frac{R_{\text{universe}}}{R_{\text{now}}}\right)^3 \sim 10^{87}\left(\frac{T_r}{M_P}\right)^3 \tag{6.6}$$

of these initial regions.

Since the scalar field has been undergoing a random walk of quantum fluctuations during the inflationary epoch, it will have a variation over these regions. Let us trace back the history of these domains in order to understand how large the fluctuations in ϕ may

[2] Sahni and Starobinsky have a comment in ref. [36] that also indicates that the stochastic nature of inflation can lead to spatial fluctuations of the cosmological constant. I thank A. Starobinsky for bringing this reference to my attention.

be. To produce this number of domains from a single domain during inflation we need N_e e-folding times, with

$$e^{3N_e} = N_{\text{regions}} \,. \tag{6.7}$$

The resulting random walk in ϕ will produce a spread in values of order

$$\Delta\phi \sim \sqrt{N_e}\frac{H}{2\pi} \tag{6.8}$$

over the visible universe.

While this will not be a uniform gradient, it provides an estimate of the variation in ϕ over the universe. In a multipole expansion a gradient term $\delta\phi_1 \sim x$ will be the leading component. Since $\nabla^2\delta\phi_1 = 0$, this component will be unaffected by the equations of motion and will survive until the present. This is converted into a variation in the cosmological constant of size

$$\frac{\Delta\Lambda}{\Lambda} = 10^{123}\frac{V'(\phi)\Delta\phi}{M_P^4} \leq 10\sqrt{N_e}\frac{H}{2\pi M_P} \,. \tag{6.9}$$

Here is the weakest part of our estimate, as we have only an upper bound on the derivative of the potential. However, this quantity cannot be too small if this model is to be applied to anthropic ideas because if the potential is too flat there will not be any influence on the cosmological constant.

Because we have only an upper bound on the variation of Λ we cannot give an unambiguous prediction. However, it is clear that under most circumstances the variation will be small. If inflation occurs at the Planck scale it is possible that there could be a visible effect, since we then have

$$\frac{\Delta\Lambda}{\Lambda} \leq 10 \,. \tag{6.10}$$

However, this bound drops rapidly with reheating temperature, resulting in

$$\frac{\Delta\Lambda}{\Lambda} \leq 10^{-9} \tag{6.11}$$

for $T_r = 10^{-5}M_P$. This mechanism does demonstrate the possibility of the variation in the cosmological constant, although its expected magnitude is likely too small to be observed. The magnitude of the effect in other mechanisms remains to be explored.

7. Conclusions

We have seen that the cosmological constant is a very sensitive probe of any potential spatial or temporal variation within the underling theory. This can be used to indirectly disfavor variations in the other parameters. If a fundamental theory does predict variations in the parameters, the cosmological constant will likely be a strong constraint. Observationally the effects of a gradient of the cosmological constant would be revealed through an anisotropy of the universe beyond our local peculiar motion.

In addition it was noted that the physics that is employed in the anthropic solution to the cosmological constant problem provides a novel motivation for searching for a variation in Λ. The magnitude of the potential signal is presently unknown, and will likely be small if the Hubble damping mechanism is correct. However, anthropic theories will be in general very difficult to test and any possible signal of such an effect deserves serious investigation.

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